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## Self-Organised Criticality at the onset of Aeolian Sediment Transport

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#### ABSTRACT



Despite decades of rigorous investigation reliable prediction of aeolian sediment transport rates continues to prove impossible. Transport rate formulae are based on the governing principal of steady state equilibrium, such that wind velocity produces a linear response in sediment flux. Field experiments however demonstrate a highly non-linear response and considerable deviation between observed and predicted transport rates. The limited predictive ability of the transport rate equations is largely attributed to crude measurement techniques that characterise wind velocity and sediment flux using time averaged values on the order of minutes effectively concealing a time scale on the order of seconds on which the equilibrium condition is established. Attempts however to resolve a characteristic time scale, continue to reveal only inextricable complexity. It is becoming clear from the study of multi-component systems that such non-linearity is a pervasive attribute of system dynamics.

Wind tunnel experiments were conducted to examine the nature of steady state sand transport under uniform forcing. Grains traversing a collimated region, illuminated by 500w halogen lighting, were acquired by video camera and transferred to computer at a rate of 10 frames per second. A suite of image analysis techniques was developed to quantify the volume of sand recorded in a sequence containing on the order of 10<sup>4</sup> images and a transport time series generated. Wind velocity measurements acquired using a Pitot tube / manometer combination and simultaneous with transport measurement, are recorded using a second video camera positioned normal to the manometer

In contradiction to the steady state hypothesis, sand transport was observed to flux sporadically suggesting that the dynamics of aeolian transport are similar to avalanches observed in a sand pile. The number size distribution for transport events shows clear power-law scaling over about 2.5 orders of magnitude which is consistent with the dynamics of self-organised critical systems. Such systems are inherently unpredictable, a fact which may contribute to our understanding of the intractability of the aeolian transport problem.

**ADDITIONALINDEXWORDS:** Self-organisation, aeolian, sediment, sand, transport.

#### INTRODUCTION

As global warming threatens expanding desertification and a loss of coastal habitat through sea level rise, research efforts in the field of aeolian sediment transport have intensified. Decades of rigorous field and laboratory studies have pursued, in particular, the problem of sediment prediction (BAGNOLD, transport rate KAWAMURA, 1951; ZINGG, 1953; HSU, 1971; LETTAU and LETTAU, 1977; WHITE, 1979; JACKSON and MCCLOSKEY, 1997). However, despite such considerable investigation reliable prediction has proven difficult and a universally applicable transport rate expression remains elusive. ARENS (1996) for example demonstrates that observed rates of transport deviate considerably from potential rates predicted by the transport rate expressions. JACKSON (1993) and SARRE (1987) show that even for the same parameter values the equations produce widely varying results. It is clear from the catalogue of formulae that exist and the diversity of form the equations assume, that our understanding of aeolian transport processes remains rudimentary.

Mass transport is generally thought to be related to an estimate of near surface wind velocity (shear velocity  $U_*$ ) and a mean grain size distribution. Despite the apparently simplistic nature of the system dynamics there exists however, a lack of consensus regarding the details of the relation. There is no agreement concerning the exponent or coefficient to be applied to  $U_*$  and the precise definition of a threshold wind velocity remains contentious (NICKLING 1988, WILLIAMS *et al.*, 1994). Several environmental

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variables, unquantified by the conventional models are also frequently cited as influencing the threshold transport rate and predictive reliability of the equations. SHERMAN andHOTTA (1990) suggest that until all the relevant contributing factors can be quantified accurate prediction will remain constrained. However, BUTTERFIELD (1991) points out that attempts to improve the models and take into account the effects of such variables still show only moderate agreement.

It is evident from such discrepancies that aeolian sediment transport is an intrinsically complex system governed by the interaction of variables operating both spatially and temporally. In an attempt to elucidate the nature of such interactions the system is reduced to its fundamental components and conceptualised in a model in which positive/negative feedback mechanisms act in a selfregulatory manner such that the system attains steady state equilibrium. The transport rate formulae derive from assumptions prescribed by the dynamics of the steady state wherein a regime of steady uniform winds blowing over a sand bed composed of a uni-modal grain size distribution produces a linear response in sediment flux. Further, experiments conducted to ascertain the field applicability of the expressions, reflect the steady state assumption on which the formulae are based. Sediment flux and wind speed are conventionally studied using data collected from single survey transects on a shore normal orientation and characterised using spatial and temporal averages. The wind field, naturally dominated by short time scale velocity fluctuations or gusts, is time-averaged over periods of minutes. Characterising the wind field at such low frequencies has forced measurements of sediment flux over similarly long periods. Unsurprisingly the correlation between actual and predicted transport rates is only moderate at best.

Recent work attributes this non-linear transport response to crude measurement techniques (BUTTERFIELD 1991; HARDISTY 1993; JACKSON 1993; JACKSON and McCLOSKEY 1997; STOUT 1998). It is suggested that averaging wind field fluctuations over time scales on the order of minutes effectively conceals the temporal adjustments to sediment flux on the order of seconds by steady state transport is established. BUTTERFIELD (1993) for example suggests that sediment flux and wind velocity histories be recorded synchronously at 1Hz frequencies or faster in order to characterise the sediment flux response at realistic time scales. LEE (1987), one of the first to suggest that the potential significance of the mass flux response at turbulent frequencies is neglected as a result of constraints imposed by the steady state that higher resolution, assumptions, concluded simultaneous measurements do not improve prediction of transport rates. However, it continues to be asserted that

until further technological advances are realised discrepancies between wind speed and sediment flux will remain unresolved (BAUER et al., 1998; STERK et al., 1998). Despite ongoing attempts to characterise the transport response at turbulent frequencies, the complexity evident in lower resolution data pervades that acquired at higher resolutions. It is, however, becoming clear from the study of complex, multi-component systems that continually emergent complexity is not necessarily the result of experimental inadequacies, but instead persists as an inherent attribute of the system dynamics.

This investigation attempts to examine the possibility that unpredictability in aeolian sediment transport may be due to non-linear interactions rather than the spatial and temporal heterogeneity imposed by environmental variables. The aim therefore was not to emulate the beach environment where gusting wind, moisture, ripple development or slope, for example, augment system complexity. Instead these experiments were designed to examine the nature of steady-state transport in a wind tunnel wherein environmental variability can be rigorously controlled, indeed eliminated; the wind velocity is maintained constant and sand sieved to ensure a uni-modal grain diameter of 0.42mm. High-resolution, digital photography is used accurately to measure sediment transport rates at a sampling frequency of 10Hz.

#### **METHODOLOGY**

Experiments were conducted in an open circuit suction type wind tunnel (test section 0.14m x 0.14m x 2m). The roof and front wall of the test section are constructed from Perspex to facilitate observation and measurement of sand transport. A 0.05m deep tray at the base of the test section accommodates the sand bed. To facilitate replenishment and levelling of the sand bed the test section roof is removable. Air and sand are expelled vertically from the tunnel via a tower diffuser resulting in a permanent sediment loss from the system. Experiments are therefore limited to ~ 30 minutes duration before the sand bed deflates and transport ceases. Air speed in the tunnel is regulated by means of an adjustable throttle valve. To examine the response of grains to wind velocity transport activity is observed over a range of increasing wind velocities. However, in an attempt to attain steady state sediment transport, wind speed is maintained constant for the duration of each experiment. Wind velocity measurements were taken simultaneous with sand transport measurements using a Pitot-tube connected to an inclined differential manometer.

Images of sand transport data were acquired using a highresolution (1024 x 1024) non-interlacing video camera mounted normal to the wind tunnel and interfaced to computer via frame grabber. A high intensity light source (500w Halogen lamp) was used to illuminate the area of the

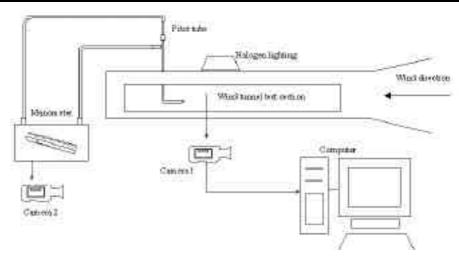


Figure 1. Experimental Configuration. Camera 1 records sand transport activity. Images are acquired and transferred to computer at 10 fps. Each data run comprises 10<sup>4</sup> images. Wind velocity measurements are taken simultaneous with sand transport measurements using a pitot tube/manometer combination. A second video camera mounted normal to the manometer recorded meniscus position and stores data on tape for later transfer to computer. Images are grabbed to computer at the same frame rate as the sand transport images.

tunnel in the camera's field of view. All ambient lighting was eliminated to minimise optical noise and improve image quality. The light source was mounted above the tunnel and the light beam projected vertically downward. The light beam, collimated along the tunnel centreline, produces a parallel beam of light which illuminates only a narrow section of the sand bed (0.05m), while the areas in front and behind remain in darkness. Using strong illumination coupled with the black interior of the tunnel and the exclusion of ambient lighting results in a strong contrast between grains transported through the collimated region and the tunnel background. By illuminating only a narrow section of the sand bed collimation improves image resolution by reducing the depth of field of the image.

To obtain quantitative information from a sequence of images regarding the nature of sand transport in the wind tunnel environment it is necessary to follow a sequence of successive image processing operations. These involve:

- 1. subtraction to remove noise
- 2. thresholding to produce binary images which facilitate unambiguous feature selection and
- 3. streak analysis to count the number of pixels corresponding to grains in transport. The equipment does not facilitate image acquisition sufficiently fast to produce images of the instantaneous position of sand grains; instead grains of sand appear as streaks.

These steps are illustrated in Figure 2.

Images are acquired in two successive steps. Initially a background image is recorded in which the light source is switched on and the tunnel remains off. Because all ambient light is excluded, only the light from the illumination source reaches the camera. An image consisting only of the light intensity emitted by the illumination source is recorded. The tunnel is then activated and the velocity adjusted until the entrainment threshold is attained. Active sand transport is recorded for a duration of ~30 minutes.

The subtraction algorithm is performed as a point operation comparing pixels in the background image with the same pixel address in each transport image. The remaining value is then assigned to the corresponding pixel location in a newly generated difference image. Subtraction removes from the transport images noise added by the illumination source and any stationary noise components such as abrasive scratches on the Perspex observation panel. This technique requires therefore that the background image and the transport images correspond pixel for pixel over their co-ordinate ranges.

Thresholding is applied, again on a pixel by pixel basis, to each of the difference images created by the subtraction technique. Each difference image (containing 256 grey levels) is converted to a binary output image in which each pixel is assigned only one of two possible values, 0 or 1. The purpose of binary thresholding is to facilitate unambiguous differentiation of the sand transport related

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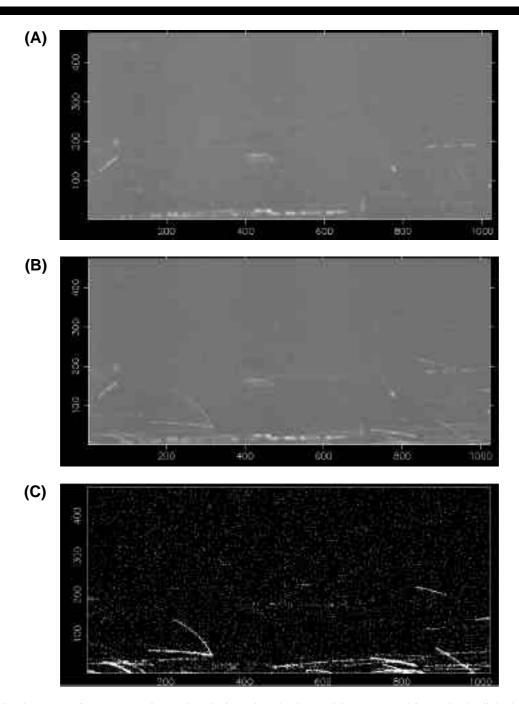


Figure 2 Sand transport image processing and analysis. A is a background image comprising only the light levels emitted by the illumination source. B is an image of sand in transport captured as streaks. Both images A and B contain bright linear features resulting from abrasion by sand. In subtracting image B from image A, this stationary noise component is removed and noise added by the illumination source reduced. Each subtracted image is then binary thresholded (Image C) and only the white pixels in each thresholded image counted to produce a record of the temporal nature of transport.

(STR) features in an image from the background. The STR features are distinguished based on the relationship of each pixel value to a manually selected threshold value. The threshold value is compared by logical operand to every

pixel in the difference image. When a pixel in the difference image satisfies the threshold condition it is reassigned the value one otherwise it is assigned the value zero.

Since each image typically contains 10<sup>6</sup> pixels and each processing stage generates a new set of images, reducing file size is necessary to preserve computer memory. This is most efficiently achieved by only storing the x,y positions of those pixels with the value one – i.e. the pixels corresponding to the STR features.

Streak analysis is a technique used to locate and measure the spatial extent of selected features whose adjacent pixels are connected and is achieved in this instance by counting the numbers of pixels which form part of the same structure (CASTLEMAN, 1996). The technique addresses each pixel in turn and searches the local pixels in a four-neighbour connectivity pattern i.e. each pixel in an image is examined to determine whether its laterally adjacent pixels (i.e. those above, below and to the right and left) are connected. Pixel connection is determined by satisfying the logical operand that adjacent pixels have the value one – indicating presence of the STR features. The operation continues iteratively until all pixels in the image are accounted for. The numbers of white pixels (those with the value one) in each image are then counted.

The count of white pixels in each image is aggregated into a transport time series, integrated into event sequences and the frequency-size distribution examined. A time series representing the temporal fluctuations in sand transport is generated (Figure 2). Transport images are grabbed to disk as quickly as possible. A total grab time is recorded at the end of a run, so that for a given number of images the average separation time can be calculated. For a run lasting 1000 seconds, in which 104 images were acquired, the average separation time is 0.100ms (or ~10 frames per second). The ordinate relates to pixel count and represents sand transport magnitude. The number of white pixels present in an image represents the amount of transport recorded at that instant in time. If a successive number of images record the presence of sand grains in transport it is taken that they form a transport event. If an image then records zero transport the preceding event is taken to have ended. Transport events are then binned according to size, and their frequency-magnitude distribution plotted in loglog space.

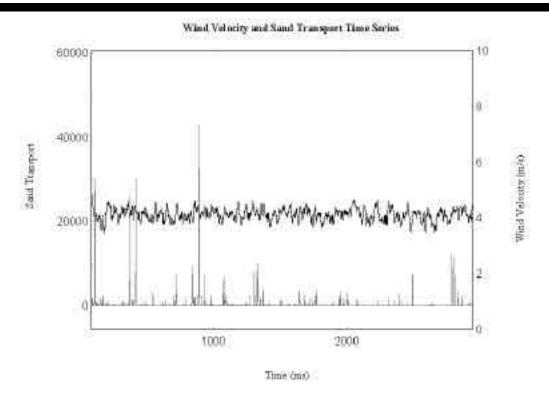


Figure 3. Comparisons of temporal fluctuations in simultaneously acquired wind velocity (top signal) and sand transport (bottom) time series. Transport event time series shows the burst/quiescent nature of the transport sequence; clusters of peaks in the sequence signify transport activity while interevent episodes record zero activity. Event size is defines as the integral of the number of white pixels counted in successive sequences of images. An individual event is considered terminated when an image is recorded in which no sand grains are apparent. Each sand transport data set comprises 1 x 10<sup>4</sup> images acquired at 10 frames per second resolution. Each point in the graph is equivalent then to 1 image acquired every 100ms. To facilitate correlation with sand transport episodes, video images of the meniscus position on the manometer were frame grabbed to computer at the same separation time. Initiation of transport events do not correlate with fluctuations in wind velocity.

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#### **RESULTS**

The most striking quality of the observed transport was its non-uniform nature in time. In direct contradiction of a steady state, characterised by a uniform cloud of grains in continuous motion, sand transport was observed to flux sporadically. Intermittent bursts of saltation were interspersed with quiescent periods during which grain motion was predominantly in the form of creep. Wind velocity data collected simultaneously with sand transport data do not reveal a correlation by which to explain this non-linear transport response; the initiation of a transport event appears to occur independent of any fluctuation in wind velocity (Figure 3). The transport event time series indicates that the size of an individual event does not depend on the size of preceding events; both large and small events are sustained and several large events are often observed in close succession. This behaviour persisted over a range of increasing wind velocities. The frequency-size distribution of transport events scale as a power-law spanning more than two orders of magnitude. This result has important implications for prediction in aeolian sediment transport and is discussed below.

#### **DISCUSSION**

These results indicate that complex behaviour emerges even in the absence of complicating external factors. Qualitatively the burst/quiescent sequences are consistent with interactions on a 'horizontal sand pile' at a critical point; here avalanches are driven by the horizontal wind momentum flux rather than gravity. The normal sand pile model serves to illustrate the concept of self-organised criticality (SOC). Devised by BAK et al., (1987), SOC proposes a unifying theory to explain the intrinsic properties of a diversity of dynamical systems with infinite temporal and spatial degrees of freedom. Stock markets (PONZI and AIZAWA 2000), ecological (UPADHYAY 2000) and geological (BAK and TANG 1989) systems have been shown to exhibit dynamics consistent with SOC. Such systems self-organise to a critical point whereat non-linear amplification of small perturbations are sufficient to trigger events of any magnitude. Importantly, self-organised critical systems do not attain a steady state but evolve between states of marginal stability (TURCOTTE 1992; MAIN 1996). According to the sand pile model, as grains are added to a sand pile at a uniform rate it evolves to a

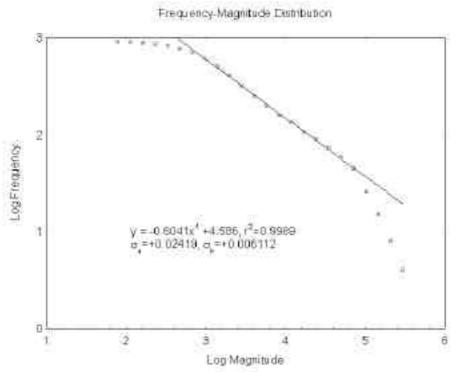


Figure 4. Sand transport event magnitude-frequency distribution. When plotted in log-log space, the frequency-magnitude distribution scales as a power-law spanning 2 orders of magnitude. Evident however from this distribution are roll-offs at high and low magnitudes. The roll of in high magnitude events represents spatial and temporal finite size effects. The largest event sustainable is subject to constraint by wind tunnel size. Further an open-circuit wind tunnel imposes permanent sediment loss from the system temporally constraining the maximum duration of an experiment. The low magnitude event roll-off suggests that small events are under represented due to apparatus constrains. The integration time used means that small events comprising small numbers of grains are not captured.

critical slope at which point the pile exhibits sensitivity to small perturbations. Each grain added to the pile will most likely come to rest, however if a grain causes the local slope to exceed the critical slope, making it unstable, an avalanche of any size or duration will be triggered. It is proposed here that in the same way that sand pile avalanches occur as a result of critical instabilities on the pile surface, sand transport events are triggered as a results of local instabilities on the sand surface. The surface configuration of the sand bed changes continuously due to the arbitrary grain impacts and creep activity by bed grains. Prior to a transport event an increase in grain bed activity is observed. The bed organises itself to a state where a single grain can have any effect on impact; it may come to rest or eject other grains triggering a transport event. Consequently unless the detailed structure of the sand bed is known at every point it is impossible to predict with any certainty when or where an avalanche will occur or what magnitude it will have. Following an episode of saltation activity, quiescence returns as the system evolves once again to the critical point. Termination of transport events is as yet more difficult to determine satisfactorily but may be the result of localised regions of critical instability so that only localised areas of the bed will be involved in a specific transport event. That sequences of both large and small events are observed and several large events can follow in succession suggests transport events do not posses a characteristic time or length scale and that an individual event does not cause relaxation over the entire surface of the

Importantly the avalanche frequency-size distribution scales as a power law; this is the quantitative signature of a system whose dynamics indicate self-organised criticality (SOC). Systems described by a power-law indicate a fractal relationship and scale invariance implying thereby self-similarity irrespective of scale; multiplicative changes in measurement scale, whether upscaling or downscaling of a system, will continue to reveal the same detail or structure. By implication then, the complexity pervasive in the aeolian sediment transport system appears not to be an artefact of the resolution at which the system is measured but is instead an inherent aspect of the system dynamics. These results may help explain the enduring unpredictability of transport rates.

#### **CONCLUSIONS**

Predicting aeolian sediment transport rates is a central focus in studies attempting to define the processes governing the dynamics of this apparently simple system. As yet, however, a predictive solution remains elusive and despite rigorous attempts to characterise the system at higher resolutions, complexity remains. To ascertain the nature of aeolian sediment transport, a series of experiments were conducted in the controlled environment provided by a wind tunnel. Essentially homogeneous conditions, represented by continuous wind velocity and a single grain size sand bed were maintained in an attempt to induce steady state sand transport. For a range of increasing wind velocities, transport was observed to flux sporadically such that episodes of active sand transport were interspersed with periods of quiescence. Such behaviour is indicative of a system governed by self-organised critical dynamics in which a steady state is not attained. That a power-law emerges for transport events in the relatively homogeneous environment provided by a wind tunnel suggests a possible explanation for the limited predictability of transport rates in the natural environment with infinite degrees of freedom. The scale invariance implied by a power-law distribution indicates that irrespective of the spatial or temporal resolution used to characterise the system, complexity will continue to emerge and predictability will remain elusive.

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